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Concept of a Stereochemical System*

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Abstract—A concept of a stereochemical system is presented based on a notion of chiral and achiral sets and on the fundamental triad of streochemistry: conformation, chirality, configuration. The interpretation of the mathematical essence of the latter is performed by application of Ruch, Noether, and Goedel theorems.

I. PREFACE

Geometric basis of stereochemistry has been obvious since the first stereochemical theory, that of tetrahedral carbon atom introduced by van't Hoff and Le Bel. The close relation of stereochemistry to the other parts of mathematics (set theory, graph theory, algebra, topology, etc.) was disclosed later. One of the earliest goals of stereochemistry was to give explanation of the optical activity. Consequently, the stereochemistry of the optically active compounds (in other words, chiral stereochemistry) became the central part of the science. It is among the most mathematically penetrated fields of chemistry, not in computational, but in conceptual aspect. Therefore it is possible to build up a logical system of stereo-chemistry. In constructing of such system two lines of development should be brought together, that of structural chemistry dealing with real molecules, and that of formal logic operating with more abstract (and more general) objects that may be applied to the world of molecules. In the ideal case the ability

should be acquired permitting the possibility to treat an actual problem from all points of view combining the structural and mathematical opportunities of analysis. Whereas the structural-chemical aspect of stereochemistry is investigated in detail and described in versatile treatises and reviews, the mathematical aspect is far less known.

In building up the system first the main concepts and the most general postulates should be assumed, and then the corollaries based thereon should be derived involving more particular objects. By "concept" we mean that here are presented the components of stereochemistry system we believe to be necessary; however they may be combined, supplemented, or, using the newfangled word, "formatted" in some other way and not only in the mode described in this paper.

Stereochemistry is a part of chemical science. Therefore it was natural that the attempt to construct a general system named "Chemistry and Logical Structures" undertaken at the beginning of nineteen seventies by a scientific team guided by Ugi [1–3] and based on permutation approach also involved the

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problems of chiral stereochemistry. The known system of the stereochemical nomenclature of Cahn–Ingold–Prelog will be characterized in the next section. An attempt to formulate a set of stereochemical axioms containing 7 principles [4, 5] includes elementary statements of the structural theory (constitution of the molecule) and also dynamic stereochemistry. It will not be considered in this paper as also the other systems describing the constitution and reactivity of molecules and not relating it to the chiral stereochemistry.

In this paper a concept of a stereochemical system will be presented based on the fundamental stereochemical triad. A special attention will be focused on the "configuration" notion, and a number of mathematical practices will be used within a scope understandable for specialists possessing a common chemical education (who also include the author). The article is conceptually related to our monograph [5] but the sum of its main statements has appeared only in the later English version of the monograph [6] and never has been published in Russian. Some supplements have been made later, and we believe that it is appropriate to publish the modern version. After the main principles related to any molecule and therefore applicable everywhere the attention will be focused on the material from organic chemistry, but the dynamic aspects including asymmetrical synthesis, and quantitative measurements of chirality will not be considered.

II. INTRODUCTION

The principal characteristic of a molecule or a substance in terms of chiral stereochemistry is optical activity. A substance can be optically active or optically inactive. Molecules can be either chiral or achiral. The chiral molecules consist of enantiomer pairs distinguished only by the angle of the optical rotation. A substance would be optically active if the fractions of the enantiomeric molecules therein are not equal. At the equal enatiomers fractions the substance is inactive (it is racemic). However no complete consistency exists in this respect between the substance and the molecules forming it. More or less exact characteristics of the individual molecules may be inferred from the studies carried out in the gas phase or in sufficiently dilute solutions. A substance built up from achiral molecules can be optically active in the solid crystalline state due to specific chirally fixed position of its molecules.

In the course of development of methods for substances investigation the principal concepts of stereochemistry were essentially supplemented. Thus the enantiomerism and diastereomerism were supplemented with enantiotopy and diastereotopy, and alongside configuration appeared conformation and chirality.

The concept of fundamental stereochemical triad (conformation, chirality, configuration) we stated in explicit form earlier [5]. A significant feature of this concept is the determination of the hierarchy of these notions (Fig.1): on the initial level is located "conformation" that is regarded as the existence form of the molecule at the given time moment; then "chirality" is introduced in order to distinguish between achiral and chiral conformations; the upper level is occupied by "configuration" which thus is not an elementary notion and requires additional explanation.
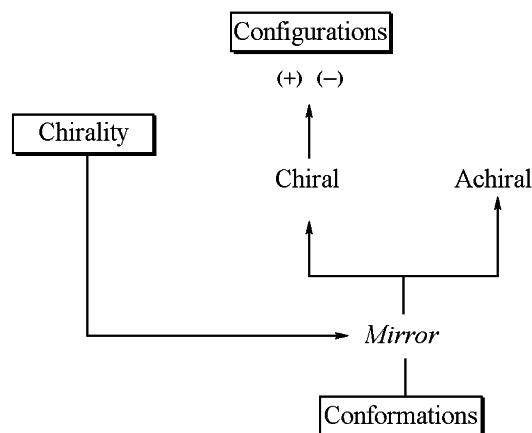

Fig. 1. Hierarchy within the fundamental stereochemical triad.

The definition of chirality is known from the works of Lord Kelvin [7] and Prelog [8] and is cited in all textbooks. The elucidation of the sense of the configuration concept proceeded throughout the last decades (see, e.g. [9–13]). The progress in this field does not mean cancellation or revision of the well-known system of Cahn–Ingold–Prelog [14] for description of stereochemistry of a molecule. However this system remains within descriptor approach where the principal goal is description of the molecule "morphology". The stereochemical morphology is characterized by a set of descriptors (indices) that are introduced according to definite rules and which should ensure unambiguous description of each chiral fragment of the molecule. The scope of these systems lies in developing detailed and consistent rules, but the understanding of the

mathematical sense of the main stereochemical notions is outside of their targets and possibilities. This is just a special section of nomenclature for the chemical substances. The target of a nomenclature consists in building up of a certain code that unequivocally reflects the structure of any individual molecule including its stereochemistry. The important novelty of the Cahn–Ingold–Prelog system was the fact that it was based on chirality concept and that it demonstrated the significance of the presence in the molecule of structural elements of chirality.

The system based on morphology studies is a necessary and useful stage of investigation. The performance of authors of Cahn–Ingold–Prelog system and of their followers operating in the same trend remind to a certain extent the work of Linneus in botany and zoology (although the scientific impact of each of these systems is not similar). The morphology description of a living body is sufficient to characterize its individual identity, but the origin of the organism, its evolution, and functions are obscure to the same extent as before. Biology required quite other studies to elucidate these problems. Summing up this part we can state that the function of descriptor systems consists in adequate accounting of the stereochemical morphology for individual molecules.

Two approaches are applied to the analysis of chiral molecules: either that of point symmetry groups or that taking into account chirality elements. Although the first approach is more rigorous, it has certain disadvantages since structurally similar molecules frequently belong to different chiral groups whereas different geometric isomers, for instance, tetrahedron and planar square MA_2B_2, fall in the same group C_{2v}. This induced Pople to develop a more complex symmetry-group notation, Framework Group Approach, that provided an unambiguous index of a molecule [15]. This approach is based on relation of the groups comprising the molecule to its symmetry elements. This principle was further applied to a system proposed for "quantitative" measurement of chirality [16].

III. CONFORMATIONAL SPACE OF A MOLECULE

Conformation is an instant picture of a molecule, its way of existence at a definite moment. It is assumed that the limits of existence of a molecule as such are real, known, and can be defined. It means that the limit is known for changes in the internal coordinates of a molecule which can occur without destroying its integrity.

The analysis of a molecule cannot be reduced to the most populated conformations for it is known that often just the low abundant conformations are more chemically active and define the reactivity of the molecule. The molecule should be presented as a full sum of its conformations. Consequently: (1) A molecule is a set of all attainable conformations. (2) The set of molecular conformations is infinite, continuous, and it comprises a conformational space (CS) or conformational diversity of the molecule.

To the CS of the molecule corresponds an infinite bound graph whose apices are conformations of the molecule, and edges show the conformational transitions. It is clear that different molecules, also structural isomers, form various overlapping CS. However stereoisomers unlike structural ones possess the same arrangement of bonds between atoms and thus they are topologically indistinguishable and should be located in a single CS. This statement corresponds to an essential possibility of enantiomers interconversion by continuous conformational transition without bond rupture. In a certain sense the stereochemistry is more exacting than topology for it finds an important difference between enantiomers based on chirality; CS is common for an enantiomeric pair (we do not discuss here the systems with topological chirality).

Let us consider now some particular cases that will show the importance of the nonobvious condition of the "attainability" of conformation. CS, or multiforms corresponding to a cyclic molecule, is a bound area including all conformations that can be attained by the molecule without rupture of covalent bonds. It is however known that at a sufficiently large size of the ring a nodal molecule may exist that is quite stable and has its own bound conformational area. With respect to the simple (isomeric) monocycle the nodal molecule is an essentially possible conformation that in no case can be attained without bond rupture, thus no conformation of the nodal molecule has a preceding state in the conformational area of the monocycle. This situation is sometimes called "Eden's Garden state". It can be described by the following expression.

$$G(m) \longrightarrow G_1(m) \cup G_2(m); \; G_1(m) \cap G_2(m) = \varnothing;$$

$$m > p.$$

Here and hereinafter are used the common notations of the set theory; $G(m)$ is the set of conformations of a monocyclic molecule (or of its topological
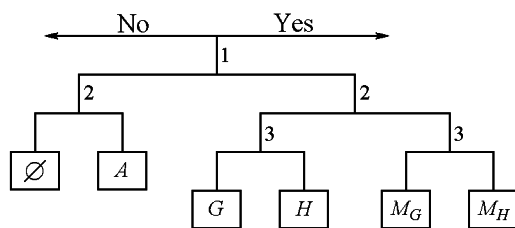
Fig. 2. Flow chart of chiral and achiral sets constructed with the use of "yes" and "no" answers to the following questions: (1) are there any chiral elements? (2) are there any achiral elements? (3) whether for each given element in this set is present the corresponding enantiomer?

isomer) containing m units; p (and further q) is a certain integer, the critical minimal value for the existence of the topological isomer (estimated for a polymethylene ring as $p = \sim 66$).

Thus at the monocycle size over the critical value the CS of the molecule is divided in two areas that have no common points and that correspond to two topological isomers. By definition [17] the real topological isomers are indispensably in the "Eden's Garden state" with respect to each other. Now let us pass from a single molecule to a population of molecules. The following expression describes catenanes appearance at decreasing size of two monocycles. Note that $p > q$.

$$G(m, n) \longrightarrow G_1(m, n) \cup G_2(m, n);$$
$$G_1(m, n) \cap G_2(m, n) = \varnothing;$$
$$m, \; n > q$$

However for rotaxanes that are "virtual" catenane analogs but with no real topological bond the equilibrium between isomers becomes possible and even is observed in some cases.

$$\text{Chain} + \text{Ring} \; \rightleftharpoons \; \text{Rotaxan}$$
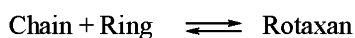
Here the conformational areas have at least one common boundary point.

$$G_1(m^r, n^c) \cap G_2(m^r, n^c) = B \neq \varnothing$$

Here indices r and c correspond respectively to the ring and the chain, and B is the boundary point or a set of such points.

Apparently a state of "Eden's Garden" can be not only of topological, but also of energetical origin: a local minimum on a potential energy surface surrounded with such a high barrier that it cannot be overcome without bond rupture.

IV. CHIRAL SETS

The necessity to introduce a general concept of chiral sets became obvious as soon as it was clear that a molecule should be considered as a sum of chiral and achiral conformations. From the very beginning Lord Kelvin defined the chirality as a maximally general phenomenon. Therefore it is quite curious that the concept of chiral and achiral sets has not been introduced till 1978 [18]. The test of mirror image is performed with each element of a certain set. If there is at least one element that is not identic to its mirror image, then it is chiral, and the set is chiral. If no such element is present, then the set is achiral. To the chiral sets belong the following subsets: racemic, containing chiral pairs, enantiomeric, containing one of elements from the chiral pair, and mixed, where achiral elements are also present. On the contrary, achiral sets cannot contain chiral elements by definition. However prochiral subsets can be isolated therefrom by another mode: their elements may be transformed into chiral ones by a single change in the structure. The achiral set can undergo further "deeper structuralization" that may be visualized by an example of alteration of ligands in a tetrahedron (see below). If more than one chiral parameter is taken into consideration (e.g., in molecules possessing several chiral fragments), then the sets are diastereomeric.

All the sets mentioned here can be put into a system by a convenient approach of building up *Flow Charts* used in classification. Flow charts are a special kind of graphs commonly used in drawing genealogical trees and evolution schemes. They were successfully used in description of symmetry groups [19]. The particular feature of these flow charts consists in putting a question at every step and placing the objects corresponding to positive answer at one side with respect to the direction of movement, and those corresponding to the negative answer at another side. At the next step another question is put, somehow adjusting the previous one, and therewith to the end of the procedure each object is placed into an individual cell, i.e. is unequivocally characterized. The classification of (monoparametric) sets with respect to stereochemistry requires but three steps (Fig. 2) with the following questions:

(1) Are there any chiral elements?
(2) Are there any achiral elements?
(3) Is there an enantiomer corresponding to each element?

Thus all 6 types of sets are classified [20]: $\varnothing$ is void set, A is achiral set, G is enantiomeric set, H is racemic set, M_G is mixed achiral-enantiomeric set, M_H is mixed achiral-racemic set.

V. ENANTIOTOPY AND DIASTEREOTOPY

Whereas notions of enantiomerism and diastereomerism were introduced in the course of the classical period of stereochemistry, the designations enantiotopy and diastereotopy appeared relatively recently. The introduction of these notions signalled the start of the new stage of stereochemistry. Just as distinguishing between two enantiomers of a molecule requires chirooptical method of investigation (measurement of the angle of optical rotation), the incomplete identity of two structurally alike atoms or groups within a molecule is revealed only with the use of nuclear magnetic resonance spectroscopy. In both cases the difference is revealed by interaction with an external (physical) agent, but in the second case the dissimilarity is finer for here is subjected to analysis the interaction between two objects (nuclei) within a molecule. Although the prochirality concept is sufficiently close to the understanding of this effect, still the decisive momentum for introduction of concepts of enantiotopy and diastereotopy by Mislow and Raban [21] was given by NMR experiment. Enantiotopy is the limit reached by enantiomerism when the difference between two groups approaches zero, and diastereotopy is the similar limit attained by diastereomerism. This fact is just reflected in the names. The enantiotopy was once [5] compared with the smile of Cheshire cat that remained when the cat proper (enantiomerism) already disappeared. This comparison fairly shows the essence of the matter. Note an nontrivial case: the diastereotopy may occur also in an achiral molecule containing simultaneously a prochiral and a pseudochiral fragments, but not two prochiral fragments. As a test serves the appearance of two chiral fragments at a single substitution, for instance, in the molecule HO–C$\underline{H}_2$–*CYR–C$\underline{H}_2$–OH.

VI. CHIRAL AND ACHIRAL SUBSETS IN CONFORMATIONAL MULTIFORMITY OF A MOLECULE

Since the conformations are "primary" and their sum provides a combined portrait of the molecule, it is clear, that CS of an achiral molecule may in general contain chiral conformations, but only as enantiomeric pairs. The chiral subset is undoubtedly void only for diatomic and triatomic molecules which are always located in one plane. For such molecules as benzene or cyclopropane the occupancy of chiral conformations possible in principle is vanishingly small, although these may exist due to insignificant deviations from the equilibrium bond lengths and bond angles. In contrast, for such molecules as hexahelicene the achiral conformations are hardly available although the thermal racemization should occur through a conformation possessing a plane of symmetry. For most of the chiral molecules due to a high energy barrier between enantiomers the achiral conformations are virtually unattainable (at temperatures lower than those where the bonds in the molecule start to cleave). However from the general point of view both enantiomers are situated in the same CS irrespective of the real possibility of racemization. The analysis of conformational multiformity of ethane or biphenyl with accounting for the rotation around the central bond [22] demonstrated that the chiral conformations of the molecule cannot be isolated points in the CS but form continuous chiral zones. On the contrary, the achiral points of CS where new symmetry elements appear, are border lines between the chiral conformation zones; they constitute prolonged achiral borders. The generalized ethane structure is a model of a "conformational rotor" [22]. An example was also cited where all the chiral elements have only achiral surrounding, and vice versa [23], but this set was discrete (the nodes of an infinite crystal lattice of the diamond type).

VII. CHIRAL AND ACHIRAL GROUPS OF SYMMETRY

From the mathematical point of view the dilemma "chirality–achirality" can be easily solved: it should be determined to which point symmetry group belongs the object (molecule). The cause of chirality in any object is its ranking among a chiral group of symmetry where the set of symmetry elements prevents the coincidence of the object with its mirror image. However the molecules with quite different structures belong to the same group of symmetry. The problem of chirality for group of symmetry was treated in detail in [5]. The symmetry criterion of chirality is as follows: *a molecule is chiral if the axis of mirror rotation symmetry S_n is lacking therein.* This general statement includes also the mirror plane ($\sigma \equiv S_1$) and inversion center ($i \equiv S_2$). Later [24] a flow chart of point groups of symmetry was introduced where the two branches, chiral and achiral, were divided already at the first step (Fig. 3).
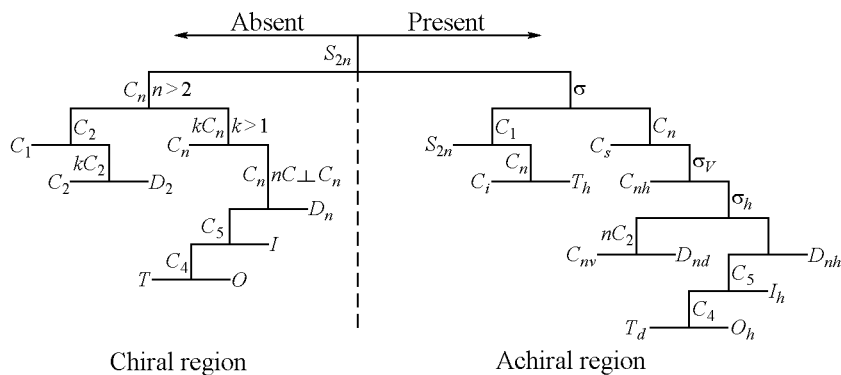
Fig. 3. Flow chart for point groups of symmetry used in chiral stereochemistry.

$$A_4 \longrightarrow A_3(B, C, D) \longrightarrow A_2(BC, BD, CD) \longrightarrow A \begin{pmatrix} B \\ R \\ S \end{pmatrix} C$$
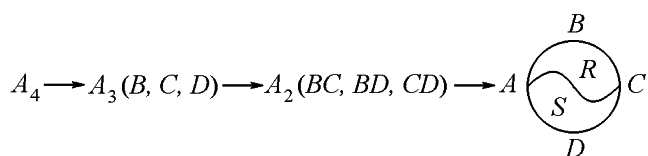
Fig. 4. A part of the complete graph of ligands substitution in methane (branch A); the existence of a methane with different ligands ABCD as two enantiomers (R and S) is symbolically indicated.
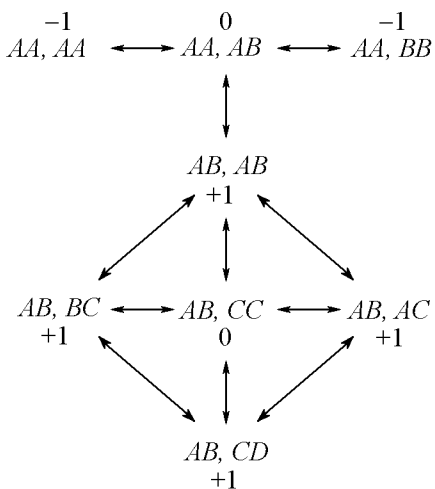
Fig. 5. A part of the complete graph of ligands substitution in allene demonstrating the transitions between chiral and achiral objects, indices of stereochemical complexity shown.

The most common coordination polyhedron in organic compounds is tetrahedron (also "truncated", with an electron pair as one of the ligands). Trigonal bipyramid arises either in a transition state during the bimolecular substitution at a carbon atom or in intramolecular coordination with a four-coordinate atom, noncarbon as a rule.

VIII. SOURCES OF MOLECULAR CHIRALITY

As was already mentioned, the chirality may appear in a single molecule or in a population of molecules, e.g., in a crystal. The structural fragments whose presence in a molecule provides its chirality were designated as *chiral elements* or *elements* of chirality; they were also called later stereogenic elements. A center, an axis, and a plane belong to this category. A helix appears in sufficiently long molecules, it is characteristic of polymers, and among compounds of low molecular weight helicenes are a classical example. Topological chirality is inherent to such molecules as knots, catenanes, Moebius strip [25].

Structural condition for a chiral center is 4 different ligands around the central atom in a tetrahedron. If the central atom in the tetrahedron would be replaced by linearly located three carbon atoms an allene structure would arise. The allen proper belongs to C_{2v} group, and its atoms are located in two mutually perpendicular planes. That is why the structural condition for a molecule with axial chirality is dissimilarity of ligands attached to each terminal atom. Two kinds of ligands are sufficient for allene chirality. Planar chirality is very rare in the molecules of organic compounds (*para*-cyclophanes) but often appears among the π-complexes of transition metals [26].

It is instructive to consider chiralization of molecules of methane and allene during introduction instead of equal ligands of three other different substituents. A complete graph of ligand substitution in methane (symmetry group T_d) contains 35 apices, among which a single one corresponds to the chiral pattern with a set of ligands *ABCD* (Fig. 4). At the same time a complete graph of allene substitution (symmetry group D_{2d}) comprises 49 apices; there appear chiral patterns of (*AB*, *AB*) type, but the cor-

responding isomeric patterns of (*AA*, *BB*) type are achiral.[*]

The part of allene graph shown on Fig. 5 demonstrates that the transition for allene from (*AA*, *AA*) to (*BB*, *BB*) is possible either along achiral route or with double crossing of the chiral border. In the methane diagram all ways not including the central apex are achiral.

IX. STEREOCHEMICAL COMPLEXITY OF MOLECULES

A problem of relative complexity of similar objects arises in various branches of science, and the solution is not always obvious. The estimation of the structural complexity of molecules and reactions in chemistry was proposed to be performed by analysis of molecular graphs [27–31]. The stereochemical complexity of a molecule can be evaluated unambiguously with an integer proceeding from an assumption that in respect of stereochemistry every molecule containing a chiral fragment is more complex than any molecule lacking such fragment [32]. This assumption is based on the fact that in going from a prochiral to a chiral object a bifurcation occurs: the molecule starts to exist as a pair of enantiomers instead of a single species in the achiral region. The following definition was suggested: *the stereochemical complexity C_s is determined by the minimum number of steps separating the molecule from its structural prochiral precursor, C_s molecule, which is assigned $C_s = 0$.*

Subset of prochiral molecules with the zero index of stereochemical complexity form the border of the achiral subset; as we go in the depth of the achiral region the C_s indices become negative, as is shown on Fig. 6 for tetrahedron (see also [32, 33]). Note on the allene diagram (Fig. 5) where another relation between achiral and chiral objects is realized the fluctuations of C_s index on the route from (*AA*, *AA*) to (*AA*, *BB*) via (*AA*, *AB*).

X. ALGEBRAIC THEORY OF CHIRALITY AND RUCH THEOREM

Ernest Ruch was among first mathematicians who treated the stereochemical problems. He demonstrated [34, 35] that all chiral molecules are divided in two classes: chiroids *a* that may form configurational

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[*] Here a convenient designation (*AB*, *CD*) was used for allene 1-*A*, 1-*B*; 3-*C*, 3-*D*.
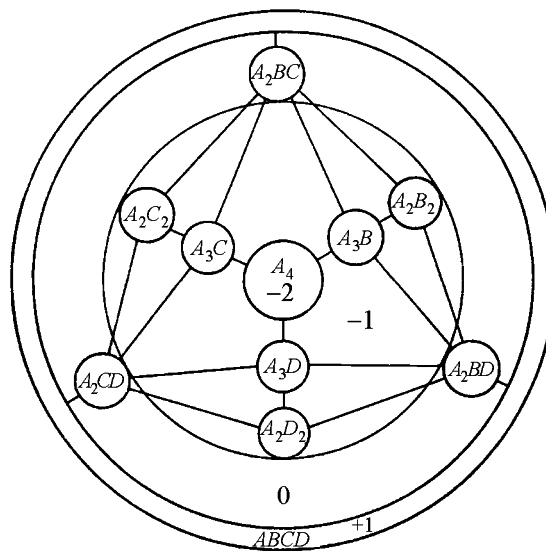

Fig. 6. The variation of stereochemical complexity indices C_s on the route from the chiral tetrahedron to the most symmetrical tetrahedron T_d (one of the four possible paths).

series, and chiroids *b* to which this notion has no sense.

The case most general for chemistry was analyzed when to an achiral molecular skeleton are added also achiral ligands resulting in a decreased symmetry and a chiral molecule formation.

Ruch theorem is as follows: the class of molecules belongs to *a* category in one of the two cases: (1) if in the molecular skeleton are present only two positions that can be occupied by ligands; (2) if the number of these positions $n > 2$, but the molecular skeleton possesses mirror planes, and in each of the planes are located $n-2$ positions available for ligands.

All the other classes of molecules belong to category *b*.

For instance let us compare two polyhedrons: trigonal bipyramid of D_{3h} symmetry, and tetragonal pyramid of C_{4v} symmetry that may be regarded as isomeric bodies with 5 apices. In each body maximum 5 places can be occupied by ligands, *n* 5. But the trigonal bipyramid has 4 mirror planes, each containing three apices (5–2), and it belongs to *a* category whereas the tetragonal pyramid has 4 mirror planes, and two of them pass through a single vertex (apical); therefore this body belongs to *b*. The octahedron and the trigonal prism fall in the same category. For organic molecules based on the tetrahedron or, more seldom, on the trigonal bipyramid, configurational correlations are carried out without complications.

Relations between dimensionality (number in parentheses) of isotropic space, chiral form filling it, and the corresponding achiral singular form

| Isotropic space	Chiral form filling the space	Corresponding achiral singular form
Straight line (1)	Sine sequence of points (0)	Multiple zero point (<0)
Plane (2)	Archimedean helix (1)	Point ("center of chirality") (0)
Space (3)	Helicoidal surface (2)	Straight line ("axis of chirality") (1)
Hyperspace (4)	Helicoidal space (3)	Plane ("plane of chirality") (2)
........., (N)[a]	........., (N–1)[a]	........., (N–2)[a]

[a] Extension to the space of any dimensionality.

XI. GEOMETRIC THEORY OF CHIRALITY

The nature of chiral elements was logically rationalized by the geometric theory of chirality [36, 37]. Inasmuch the chirality is a general phenomenon it is feasible for a while to turn from actual molecules to infinite chiral structures. For better understanding of chirality let us put the following question: what should be the form of chiral bodies which would fill an isotropic continuum with no gaps and overlapping? In other words, we should find a chiralization mode of isotropic straight line R^1, plane R^2, Euclidean space R^3, hyperspace R^4. The existence of chiral helicene molecules that are fragments of spiral (helicoid) surface occurring in the molecular matter suggests the way to total filling of isotropic space R^3 with a two-dimensional figure. Parametric equations of a helicoid are like follows.

$$x = a\cos\varphi, \ y = a\sin\varphi, \ z = F(a) + b\varphi \ (-\infty < a < +\infty).$$

If a random straight line normal to some fixed axis will slide along this axis according to the above law, with $b \to 0$ and $a \to \infty$, then the space would be filled with the forming helicoid surface. The sense of chirality, i.e. the absolute configuration of the helix, is determined by the direction of sliding, thus by the sign of b. Similarly the plane R^2 is chirally filled with Archimedean helix: $x = a\eta\cos\varphi, \ y = a\varphi\xi\sin\varphi$. A
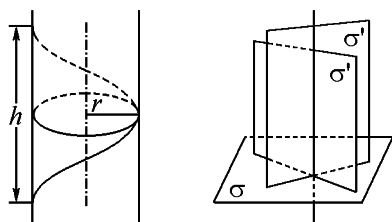
Fig. 7. Relation between elements of chirality and dependence of plane of chirality position on the operation of the molecule dechiralization.

chiral sequence of points placed according to sine law on a straight line is a logical extension of this trend to R^1 as was shown before [36, 37].

The chiral filling of isotropic continuums R^n has one very important feature: the process of filling n-dimensional space with a chiral $(n-1)$-dimensional form requires an obligatory presence of a singular achiral $(n-2)$-dimensional form on which the filling form is wound up. For instance, the cylindrical helicoid fills the space R^3 using a singular straight line as an axis. and the Archimedean helix fills the plane R^2 using a singular point as a center of the helix. The extrapolation of this rule to the four-dimensional space R^4 requires the presence at its chiral filling with the three-dimensional space of a two-dimensional plane as a singular element of the whole system. It should be stressed again that all these singular elements of the chiral systems are achiral. The general law summed up in a table is as follows: n-dimensional isotropic space is completely filled with $(n-1)$-dimensional chiral form possessing as a "singular element" (n-2)-dimensional achiral form.

Just these achiral "singular elements" of lower dimensionality in chiral systems of higher dimensionality are called in stereochemistry "elements of chirality" or "chiral elements". The latter name is a sheer nonsense since all these elements are a fortiori achiral. The adjective "chiral" is used here as a sign that the element is related to chiral structures of higher dimensionality where it is an essential and even indispensible part. It should be specially indicated that a "chiral plane" is an achiral element of a chiral structure in the hyperspace R^4 which thus for the first time appears in a natural way in a chemical reasoning.

We have previously described [36, 37] the transition from circular helix or helical surface to conical ones, projecting of forms from the space R^4 into the space R^{n-1}, and how the elements of chirality result

from the successive convolution of the chiral space with reduction of dimensionality. The ambiguity of the chiral plane (in contrast to the center and the axis) has been also rationalized: its position depends on the implied operation of the molecule dechiralization (Fig. 7).

XII. MATHEMATICAL SENSE OF A STEREOCHEMICAL CONFIGURATION

Thus in the general sense the CS of a molecule is comprised of a number of chiral and achiral zones; therewith the chiral zones for chiroids of Ruch class *a* have achiral borders. If the achiral zone is continuous, i.e. the chirality function does not change sign within the zone, then according to Noether theorem there is an invariant in this subset. The chirality plays here the role of a function, and it may have only one of the three values: $+1$, 0, -1. The function becomes zero at the border of the zone. The Noether theorem in physics is responsible for existence of functions of state that characterize the state of the system and do not depend on its past history. In the stereochemistry this theorem also requires *existence of an invariant, incident to the continuous zone of chiral conformations with an achiral border; just this invariant is a configuration.*

The stereochemical configuration conforms to the characteristics of function of state: if the chiral zone is not continuous, then within it the sign of chirality function changes by a jump, not passing through zero, i.e. the homochirality condition does not hold. and therefore the chiroid does not belong to *a* class, but to *b* class.

Thus the configuration is placed at the crossing of operation of three mathematical theorems [6]: (1) Noether theorem establishes the fact of invariant existence that is incident to the continuous zone of chiral conformations which has an achiral border; (2) Ruch theorem indicates the types of molecular structures that occur in such zones; (3) Goedel theorem evidences that configuration cannot be established with methods inherent only to the chemistry, but it is possible for procedures of another branch of science (actually, for X-ray diffraction analysis).

XIII. ANALYTICAL EXPRESSION FOR THE STEREOCHEMICAL CONFIGURATION

Inasmuch the stereochemical configuration in keeping with above may be regarded as occupying the highest level of the stereochemical hierarchy whereas the conformation is on the lowest level, a problem

may be set of looking for an explicit analytical expression of conformation as a function of one or several structural arguments [38]. Let us designate the configuration as $A©$. This symbol was chosen as abbreviation of *absolute configuration*, the emphatic synonym of *stereochemical configuration.*

Since the chirality problem has been transferred from the level of molecules to the level of conformations considered in the assumed treatment as more elementary notion than molecule, the problem of chirality of a molecule as a sum of all its conformations requires an adjustment. Evidently the following conditions may be written for molecules:

$$\text{chiral}\quad \sum_{i=1}^{\infty} A©_i \neq 0, \quad \text{achiral}\quad \sum_{i=1}^{\infty} A©_i = 0.$$

This expression is a formalization of a fact that in the conformational multiformity of an achiral molecule for each chiral conformation, if any, can be found the corresponding enantiomeric one; thus the conformational multiformity of any achiral molecule belongs to the set either A, or H, or M_H, whereas that of a chiral molecule belongs either to the set M_G or G (see above).

As known from Ruch theorem [35], stereochemical configuration with respect to validity of configurational series is meaningful only for molecules possessing homochirality. This characteristic is inherent only to two principal polyhedrons: a tetrahedron, and a trigonal bipyramid (and also a distorted tetrahedron, disphenoid of D_{2d} symmetry). This can be written in quantifier form as follows [24].

$$\forall M_\chi \ \{M \in T_d^4,\ T_{3h}^5\} \ \exists\ A©$$

This expression means that *for every chiral molecule with a skeleton belonging to a point group of symmetry of tetrahedron or trigonal bipyramid exists an absolute configuration.*

In this expression the superscript at the symbol of the point group shows the number of ligands attached to the central atom. The useful supplement just in this case allows distinguishing the trigonal bipyramid from the trigonal prism that unlike the former does not possess the homochirality.

The function of absolute configuration $A©$ can have three values: $+1$ and -1 for enantiomeric objects (chiral molecules or conformations, and 0 (zero) for achiral molecules and conformations.

For molecules possessing any element of chirality, and also for homochiral polyhedrons the following
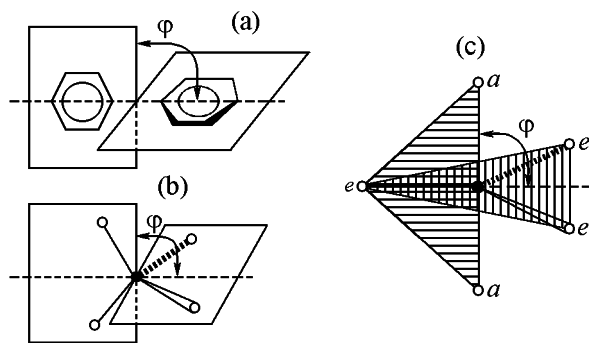
Fig. 8. Angle φ, defining the configuration $A©$ of chiral biaryl (*a*), tetrahedron (*b*), or trigonal bipyramid (*c*).

analytical expression is valid for configuration as a function of a single structural parameter:

$$A© = \text{sgn}[\sin\varphi],$$

where φ is a dihedral angle between planes containing each a pair of ligands; in the trigonal bipyramid one of the planes contains three ligands. The determination of angle φ is clear from Fig. 8.

XIV. CONCLUSION

This article was based on our proper approach to the problem, but the results of the other authors were integrated wherever possible. Anyone who tried to construct a logical system knows very well that it is difficult to include therein correct results obtained in the other systems without violating the logic and coherence of description. Just here lies the main difficulty in development of the generally recognized concept and the reason of objections that some authors have against the results of others. It is difficult to combine, add to the proper system the useful parts from the work of the other researchers. However there are some obligatory things that cannot be omitted, for instance, Ruch theorem or nomenclature of Cahn–Ingold–Prelog; these should be indispensible parts of every stereochemical system. Hopefully, to a certain extent we succeeded in such integration.

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